

Comment on ‘Controversy concerning the definition of quark and gluon angular momentum’ by Elliot Leader (PRD 83, 096012 (2011))

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It is argued by the author that the canonical form of the quark energy-momentum tensor with a partial derivative instead of the covariant derivative is the correct definition for the quark momentum and angular momentum fraction of the nucleon in covariant quantization. Although it is not manifestly gauge invariant, its matrix elements in the nucleon will be non-vanishing and are gauge invariant. We test this idea in the path-integral quantization by calculating correlation functions on the lattice with a gauge-invariant nucleon interpolation field and replacing the gauge link in the quark lattice momentum operator with unity, which corresponds to the partial derivative in the continuum. We find that the ratios of three-point to two-point functions are zero within errors for both the u and d quarks, contrary to the case without setting the gauge links to unity.

There is an ongoing controversy in QCD over the issue of how to separate total angular momentum between the quark and glue sectors and, moreover, whether there exists a gauge-invariant decomposition of the glue angular momentum into spin and orbital angular momentum components as in the quark case. While several versions of gauge-invariant separation of the glue spin and orbital angular momentum have been proposed [1–4], X. Ji argued that such a separation cannot be achieved [5]. On the other hand, E. Leader asserts [6] that, if the momentum operators are to be the generators of spatial translations, then the gauge-invariant Bellinfante momentum operators are not correct and that rather it is the canonical form that is correct.

Since this approach is based on covariant quantization, we shall use the path-integral quantization to check this conclusion. In the Monte Carlo simulation of QCD on the lattice, the number of degrees of freedom is finite, and one does not need to fix the gauge. In this case, it is usually presumed that gauge-variant observables vanish according to Elitzur’s theorem. However, E. Leader suggests [7] that, despite the fact that the operator is not manifestly gauge invariant, there could be a gauge-invariant part of the matrix element. To check this possibility, we perform a lattice gauge calculation of the quark energy-momentum tensor operator. To calculate the quark momentum fraction in the nucleon on the lattice, one typically considers the ratio of the 3-point to 2-point function

$$\frac{1}{p_i} \frac{\text{Tr} \Gamma G_{NON}(t_f, t, t_0; p_i)}{\text{Tr} \Gamma G_{NN}(t_f, t_0; p_i)} \xrightarrow{t_f - t \gg 1, t - t_0 \gg 1} \langle x \rangle, \quad (1)$$

where Γ is the projector onto the unpolarized nucleon state. The two-point and three-point functions for the

nucleon are

$$G_{NN}^{\alpha\beta}(t_f, t_0; p_i) = \sum_{\vec{x}} e^{-ip_i(x-x_0)} \langle \chi^\alpha(\vec{x}, t_f) \bar{\chi}^\beta(\vec{x}_0, t_0) \rangle, \quad (2)$$

$$G_{NON}^{\alpha\beta}(t_f, t, t_0; p_i) = \sum_{\vec{x}_f, \vec{x}} e^{-ip_i(x_f - x_0)} \langle \chi^\alpha(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \bar{\chi}^\beta(\vec{x}_0, t_0) \rangle, \quad (3)$$

where χ is the nucleon interpolating field and t_0/t_f is the source/sink time of the nucleon interpolation field. The lattice operator \mathcal{O} for the momentum that has been calculated in the literature [8] is

$$T_{4i}(x) = \frac{-i}{8a} \frac{1}{2\kappa} [\bar{\psi}(x) \gamma_4 U_i(x) \psi + \dots] \quad (4)$$

with $U_i(x)$ being the gauge link. This corresponds to the continuum tensor operator for the quark momentum $P_i = \frac{-i}{4} \bar{\psi}(\gamma_4 \overleftrightarrow{D}_i + \gamma_i \overleftrightarrow{D}_4) \psi$ which is of the Bellinfante form. E. Leader argues [6] that the correct momentum operator should be the canonical form, with the covariant derivative in the Bellinfante form replaced with the partial derivative. This corresponds to setting the gauge link U to unity in the lattice operator $T_{4i}(x)$ in Eq. (4).

We performed a lattice calculation on a $2+1+1$ -flavor $24^3 \times 64$ HISQ lattice [9] (HYP-smearing) with the Wilson-clover valence fermions. The pion mass is 310 MeV with lattice spacing around 0.12 fm, and 127 configurations (each with 4 sources) were used in the calculation. The nucleon source is shifted to $t_0 = 0$ and sink timeslice is at $t_f = 12$. The ratio in Eq. (1) for the connected insertion is plotted in Fig. 1 as a function of the operator insertion time t for the u quark in the upper panel and the d quark in the lower panel. The results with the Bellinfante form are shown as blue circles; they are obviously non-zero. The quark momentum fraction $\langle x \rangle$ is usually extracted from a plateau in a window away from the source and the

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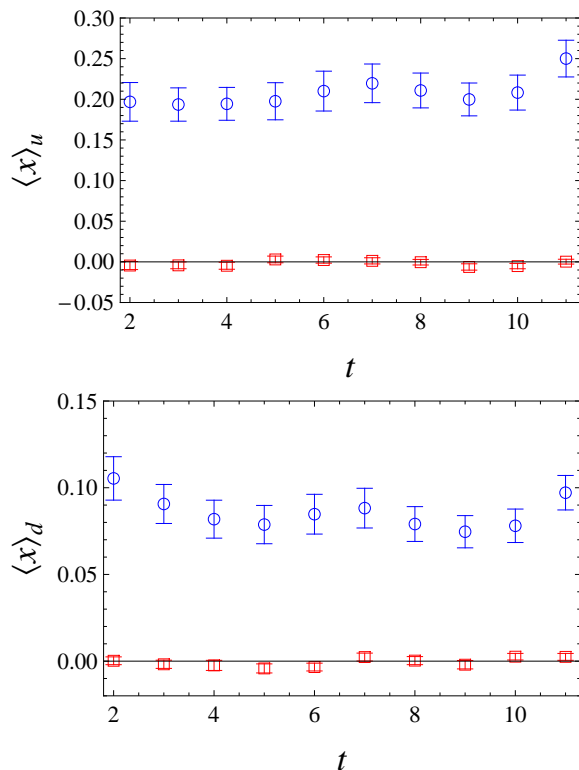


FIG. 1. The ratio of the three-point to two-point functions in Eq. (1) is plotted as a function of the operator insertion time t for the u/d quark in the upper/lower panel. The blue circles are results from the momentum operator with the covariant derivative and the red squares are those corresponding to the partial derivative.

sink. On the other hand, when the gauge link U is set to unity, which corresponds to a partial derivative, the results (red squares) are zero within errors for practically all insertion times t . We conclude from this calculation that the canonical form of the quark energy-momentum tensor with partial derivatives does not lead to non-zero matrix elements in the nucleon.

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